

# Fundamental theories in a phantom universe

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## Abstract

Starting with the holographic dark energy model of Li it is shown that the holographic screen at the future event horizon is sent toward infinity in the phantom energy case, so allowing for the existence of unique fundamental theories which are mathematically consistent in phantom cosmologies.

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The existence of the holographic bound in entropy [1] is manifested also in accelerating cosmology in the form described in the so-called holographic dark energy models. In fact, recently Li [2] has proposed a model of holographic dark energy where the quantum idea that the short distance cut-off is related to the infrared cut-off [3] is used by assuming that the latter cut-off is the size of the observer-dependent future event horizon. This choice allows for a dark energy cosmological scenario and also for an equation of state of the resulting holographic model which can be compatible with present observational constraints [4].

If we assume dark energy domination, then the Li model for holographic dark energy [2] leads to key expressions for the Friedmann equation and the size of the future event horizon which are given by

$$H^2 = 8\pi G\rho/3 = c^2 L^{-2} \quad (1)$$

and

$$L = R_h = a \int_t^\infty \frac{dt}{a}, \quad (2)$$

where  $H = \dot{a}/a$  is the Hubble parameter, with  $a \equiv a(t)$  the scale factor,  $\rho$  is the dark energy density,  $c$  is a numerical parameter,  $L$  is the size of a ultimate region whose total energy was chosen not to exceed the mass of a black hole with the same size, and  $R_h$  is the size of the future event horizon. Using Eqs. (1) and (2), with  $L = R_h$ , Li showed [2] that the index of the perfect-fluid equation of state  $w = p/\rho$ , where  $p$  is the pressure, can be written as

$$w = -\frac{1}{3} - \frac{2}{3c}, \quad (3)$$

in which the parameter  $c$  can generally take on values  $\geq 1$  and also  $< 1$ . If  $c > 1$  then we have a holographic quintessence model, if  $c = 1$  we obtain a cosmological description that corresponds to the existence of a positive cosmological constant, and

if  $c < 1$  we obtain a phantom cosmology model [5]. Although Li argued in favor of the cosmological constant case in Ref. [2], any value of  $c$  is in principle allowed by the model.

In this short letter we shall show that whereas the size of the future event horizon allowed by the Li model is always finite for  $c \geq 1$ , for  $c < 1$   $R_h$  should become necessarily infinite, contrary to the claim of Li himself [2]. Our argument is almost immediate. We start with the general expression for the dark energy density derived from the conservation law for cosmic energy,  $\rho \propto a^{-2(1-1/c)}$ , which, when inserted into the first equality in the Friedmann equation (1), leads to a general expression for the scale factor given by

$$a = \left( a_0^{1-1/c} + (1 - \frac{1}{c})(t - t_0) \right)^{1/(1-1/c)}, \quad (4)$$

with  $a_0$  the value of the scale factor at the initial time  $t_0$ . Although an initial value of the scale factor can be set equal to 1, for the sake of generality we shall keep an arbitrary value for it. Thus, inserting Eq. (4) into Eq. (2) and integrating, we obtain for the future event horizon the general expression

$$R_h = -cT^{1/(1-1/c)} \left( T^{-1/[c(1-1/c)]} \Big|_t^\infty \right), \quad (5)$$

where

$$T = a_0^{1-1/c} + (1 - \frac{1}{c})(t - t_0) \equiv a^{1-1/c}. \quad (6)$$

It can first be checked that the holographic expression  $H^2 = c^2 R_h^{-2}$  is in fact satisfied when we use Eqs. (4) and (5) for any value of parameter  $c$ . Next, we can readily see that if  $c > 1$  then  $R_h = cT$ , but when we allow  $c < 1$ , then inexorably  $R_h = \infty^1$ . Thus, the holographic phantom energy model directly constructed from the general holographic dark energy model of Li by simply imposing the condition  $c < 1$  sends the future

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<sup>1</sup>Note that  $T$  becomes negative after the big rip [8] at  $t_* = t_0 + [a_0^{1/c-1}(1/c - 1)]^{-1}$

event horizon towards infinity<sup>2</sup>. This result has an immediate implication in string and M theories. In fact, it follows from the above discussion that whereas the puzzle about how a fundamental theory can be formulated within a finite box in accelerating cosmology [6] remains operative in holographic quintessence or cosmological constant models, such a puzzle automatically disappears in holographic phantom cosmology as all the space regions are in this case fully accessible to light probes and therefore the S-matrix or S-vector description required in string or other fundamental theories could perfectly be mathematically formulated.

We interpret this result as follows. The question of how a fundamental theory can be constructed within the finite region enclosed by the future event horizon of de Sitter and quintessence cosmological models has been regarded to be a consequence of the so-called cosmological complementarity [7] from which the wave/particle duality or the superposition principle of quantum mechanics are thought to be nothing but mere limiting cases. According to the cosmological duality principle all objects in the universe have two aspects of their existence, on the one hand they can behave as members of the whole cosmic collective and, on the other hand, such objects can also behave as local individuals in the universe, so that all these objects ought then to be defined by pairs of numbers, each pair specifying the value of the dynamical physical variable describing the state of the object. It appears clear that the algebra of

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<sup>2</sup>Since at the big rip there is a real singularity it could be thought that the whole universe after the big rip would be inaccessible to current observers, as one should cut the infinite hypersurface at  $t = t_*$ , leaving the universe at  $t > t_*$  disconnected from such observers. However, sufficiently grown up Lorentzian wormholes are made possible in the phantom regime which may connect the regions before and after the big rip to one another [9], so rendering connected the whole spacetime. Had we cut off all the space-time after the big rip, then the region with size  $L$  ought to be replaced for a new horizon at time  $t = t_*$  in the future, defined by

$$R_* = a \int_t^{t_*} dt/a = cT = R_h(c < 1) < R_h(c \geq 1),$$

which, at first sight, would even aggravate the event horizon puzzle relative to the quintessence and cosmological constant cases. Thus, if no wormholes were allowed to occur, an interpretation in terms of cosmological complementarity would again be required (see later on). Becoming the accessible universe then infinite though will make any fundamental theories based on a S-matrix or S-vector formulation to be consistently defined mathematically.

complex numbers may account for this, whereby the real component is associated with the collective gravitational aspect and the imaginary component is associated with the individualized aspect describing the forces that have to be accommodated within a common gauge theory. In the accelerating cosmological scenario with  $c \geq 1$  there can then be many S-matrices of fundamental theories which are related to each other by gauge transformations and physically correspond to the superposition implied by the principle of cosmological complementarity. Now, the emergence in the future of a big rip singularity in phantom cosmologies [8] (see however Ref. [10]) breaks this duality, leaving only a cosmic behavior for all matter in the universe, at least as one approaches the singularity. It follows that in phantom cosmologies showing a big rip singularity in the future and allowing for growing Lorentzian wormholes<sup>3</sup> it is possible to have a unique mathematically well-defined S-matrix or S-vector description and hence a unique fundamental theory. It is for this reason that in the field theory description of a phantom fluid, one has necessarily to introduce a Wick rotation of the phantom field which converts this field into a gravitational object, and makes negative the scalar-field kinetic term [11]. Such negative kinetic terms are though the ultimate responsible for the the existence of the violent instabilities found for the phantom fluids [12]

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<sup>3</sup>The main difficulty which has been encountered for allowing the existence of wormholes is the quantum instability produced by catastrophic vacuum fluctuations near their throat [13]. Nevertheless, if as one approaches the big rip singularity the quantum-mechanically sustained microscopic, local behaviour of any objects is being erased off, then the wormholes would stabilize near the singularity and the universe can whereby become connected throughout its entire evolution

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